Evaluating Neutrino Moment Closures in Neutron Star Mergers

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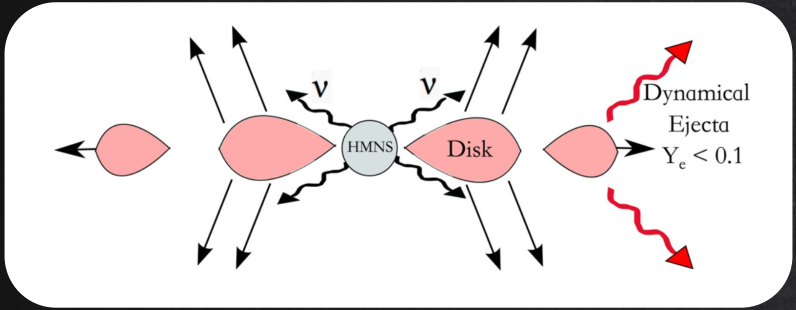
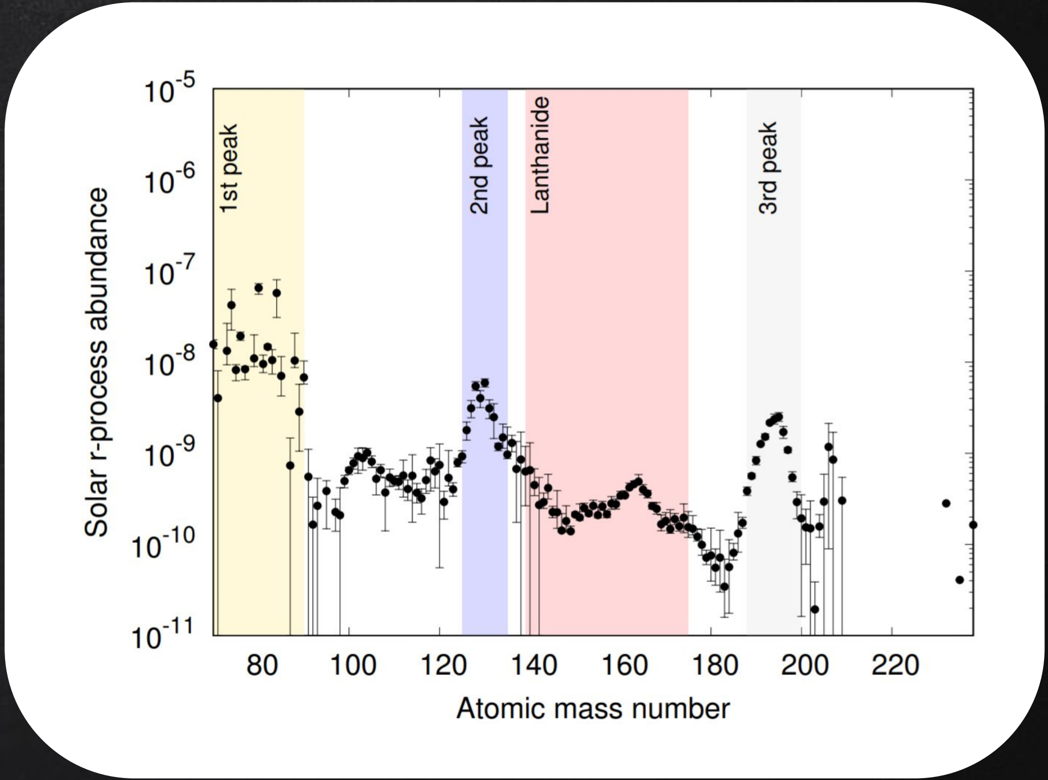
Neutrinos determine dynamics & nucleosynthesis in mergers



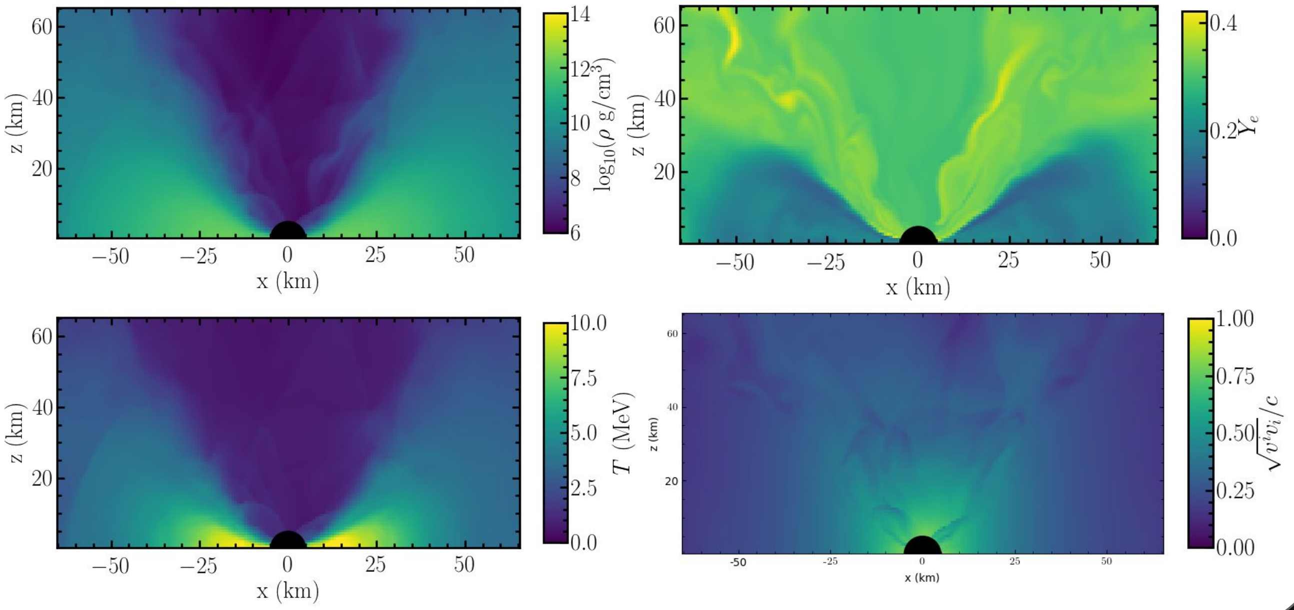
Metzger & Fernandez (2014)



Neutrinos can:

1. **Cool the disk**
2. **Drive outflows**
3. **Protonize outflows**

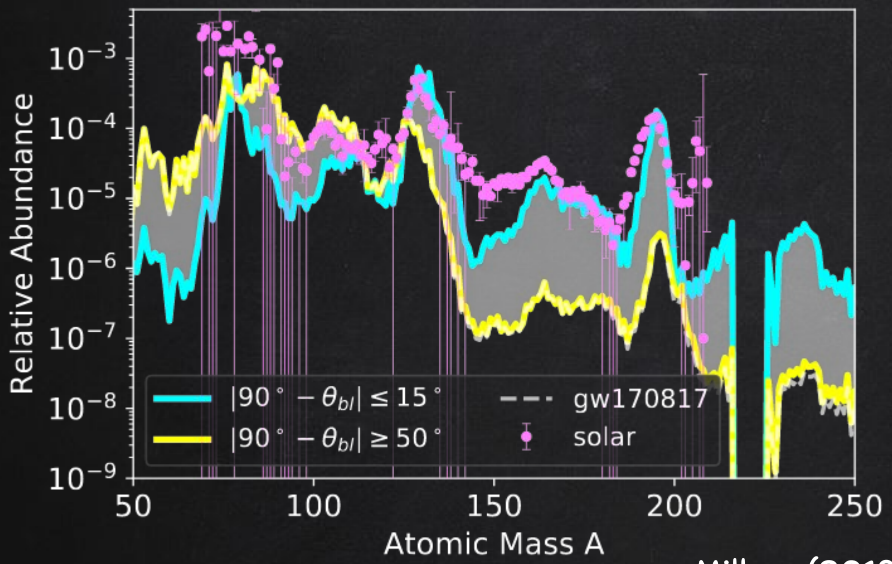
31ms After Merger

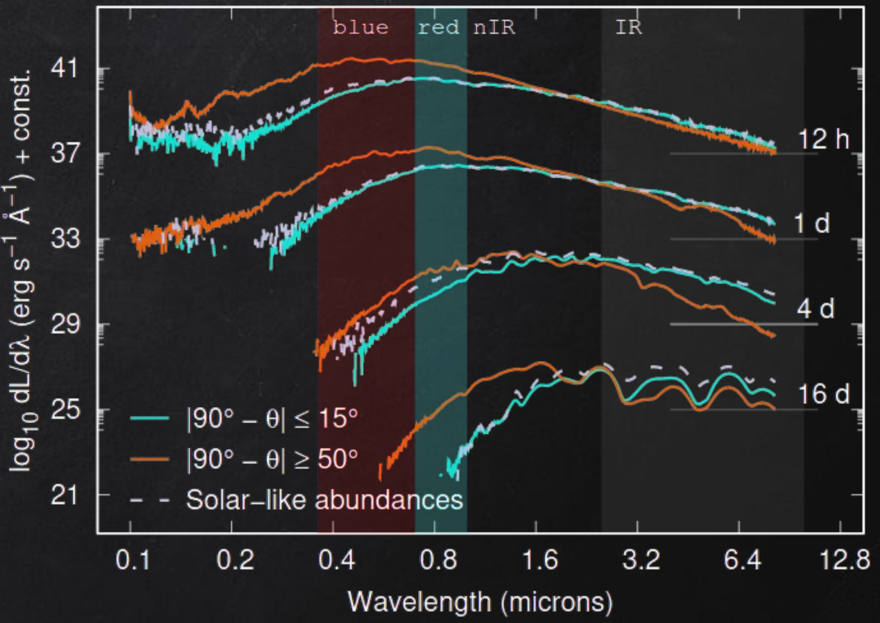


(Data from Radice+ 2018)

How/where are the neutrinos transporting leptons and energy?

Monte Carlo GRMHD Simulation

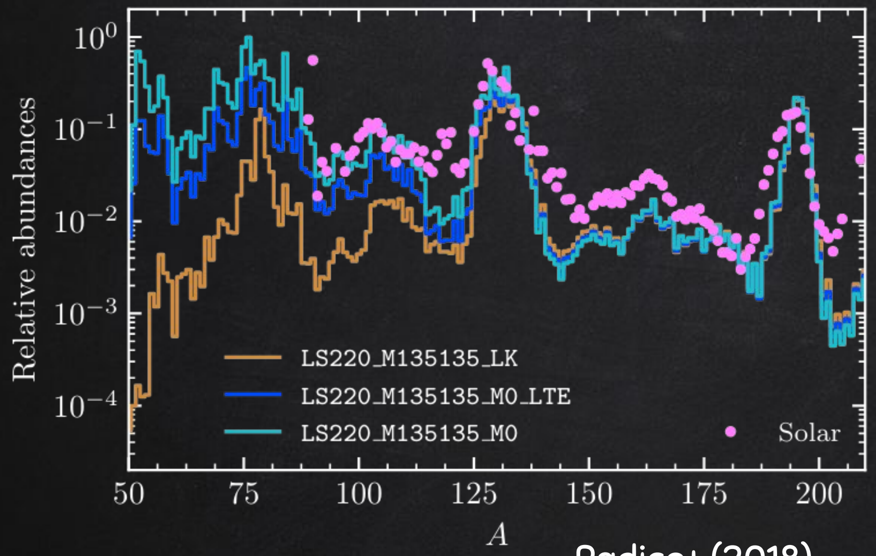




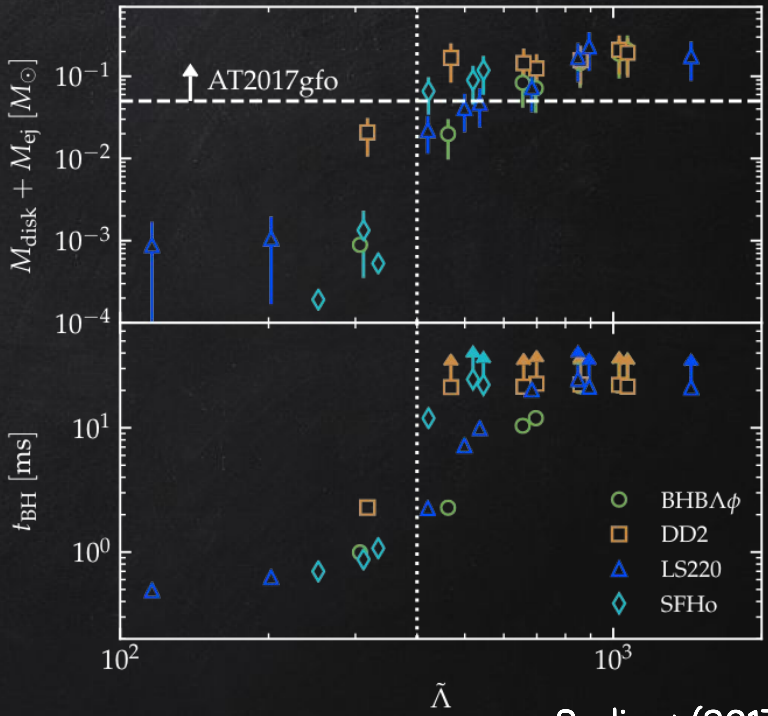
Miller+ (2018)

Accurate simulations can be matched to observables.
But are expensive.

Parameter studies require many simulations



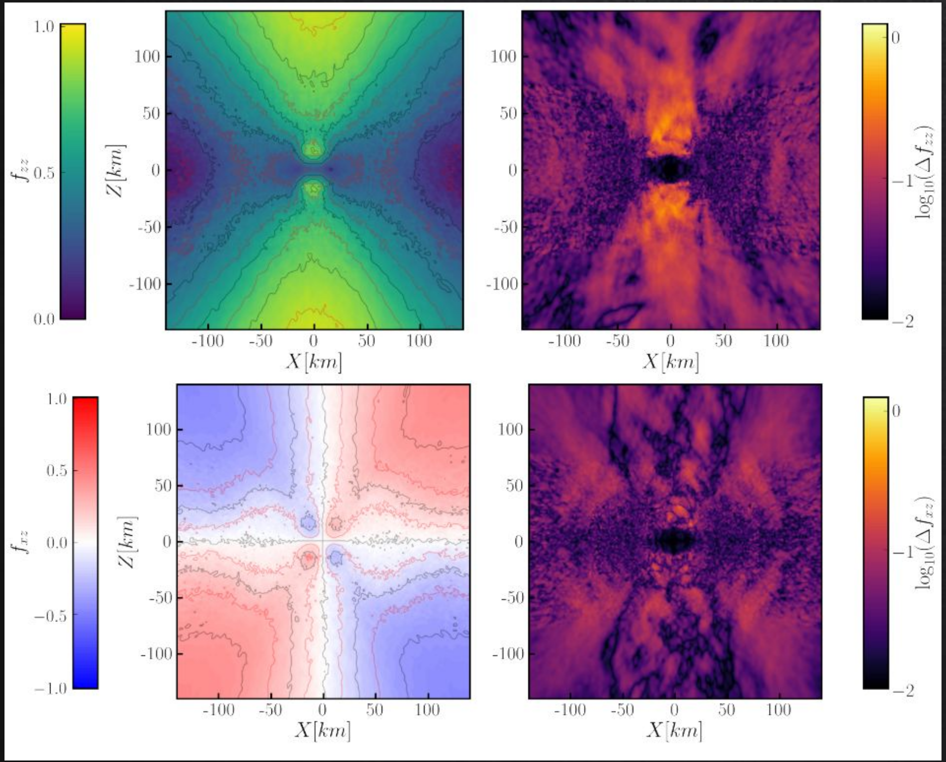
Radice+ (2018)



Radice+ (2017)

But approximations induce errors.

Exact methods can improve approximate methods

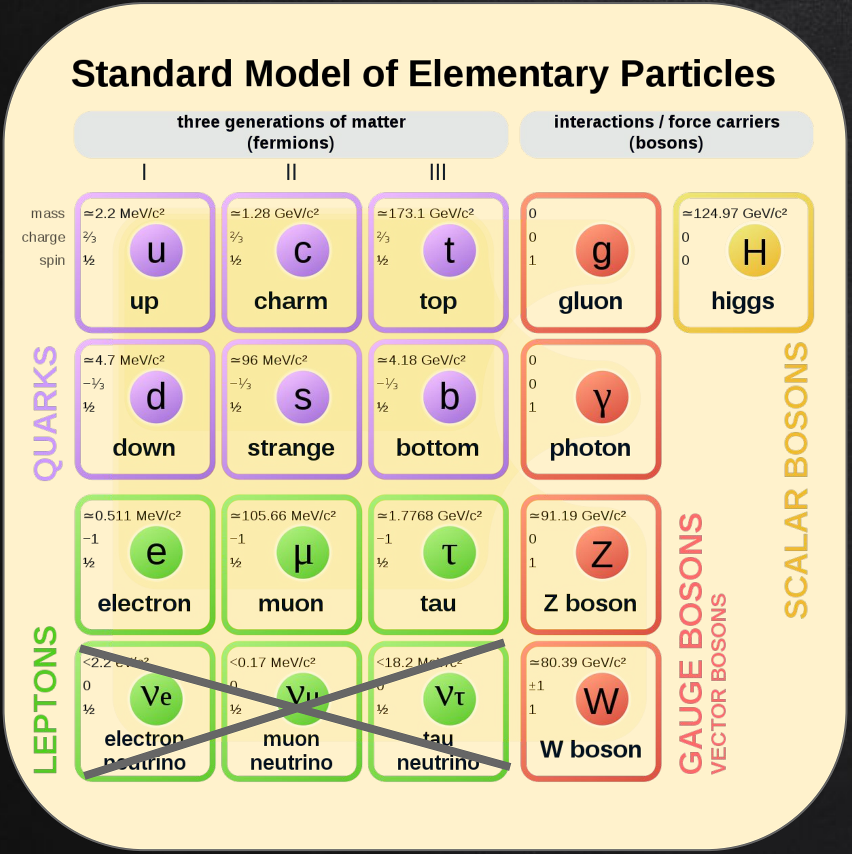


Foucart+ (2018)

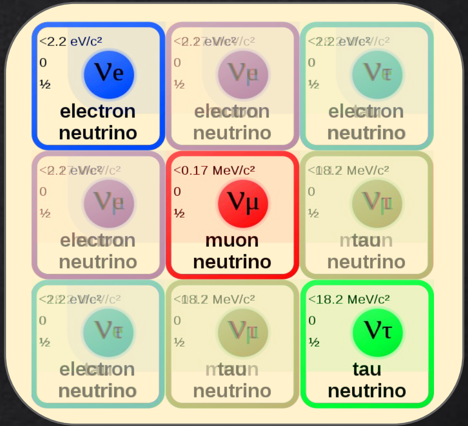
Moment-based results *qualitatively* match Monte Carlo results.

How is transport simulated?

A Deeper Look at the Neutrinos

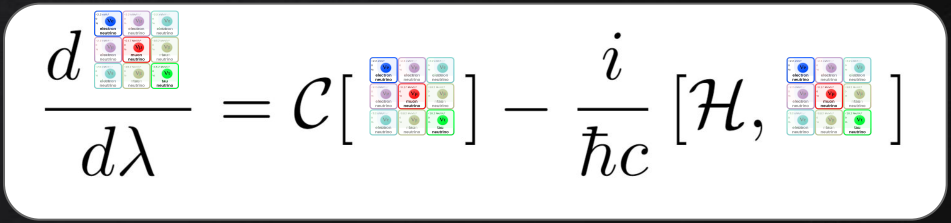


But the neutrino flavors are mixed!
(Pontecorvo 1968, Wolfenstein1978, Mikheev & Smirnov 1985)



Cardall 2008
Vlasenko et al. 2014
Blaschke+Cirigliano 2016

Move Collide Transform

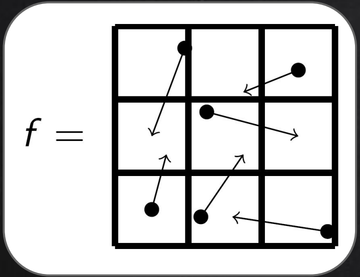


Neutrino Transport

(How many neutrinos are where and moving in what direction?)

$$\frac{d\ f}{d\lambda} = \mathcal{C}[\ f\ ]$$

Monte Carlo



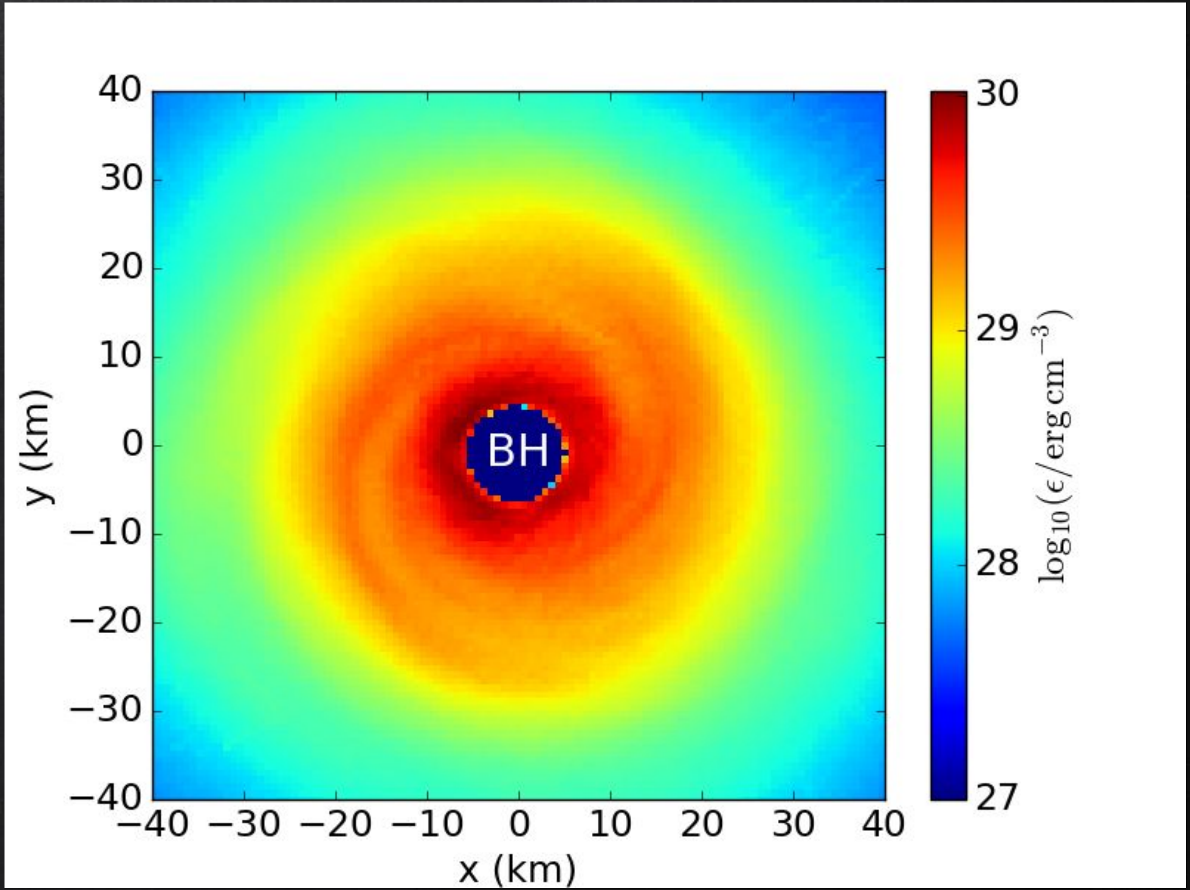
$$\frac{dk^\alpha}{d\lambda} = -\Gamma^\alpha_{\mu\nu} k^\mu k^\nu$$

$$E_{\mathrm{rad},q} = \bar{N}_q ds_{\mathrm{move}} k^t_{\mathrm{tet}} / \mathcal{V}_i .$$

SedonuGR

github.com/srichers/SedonuGR

- 3D background
- Spawn in **random** location...
- Move in **random** direction...
- For a **random** distance…
- Scatter to a **random** direction and energy
- Continue until the neutrino escapes
- Repeat 10^12 times



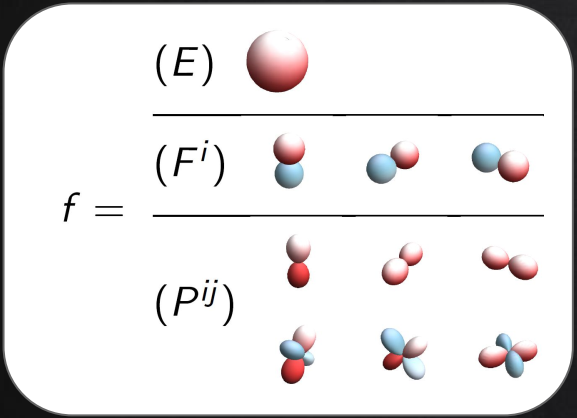
3D general relativistic calculations of Boltzmann neutrino transport.

Neutrino Transport

(How many neutrinos are where and moving in what direction?)

$$\frac{d\, f}{d\lambda} = \mathcal{C}[\ f\ ]$$

<u>Two-Moment</u>



$$\partial_t(\sqrt{\gamma}E_{(\nu)}) + \partial_j[\sqrt{\gamma}(\alpha F^{\ j}_{(\nu)} - \beta^j E_{(\nu)})] + \frac{\partial}{\partial\nu}\Big(\nu\alpha\sqrt{\gamma}n_\alpha M^{\ \alpha\beta\gamma}_{(\nu)}\nabla_\gamma u_\beta\Big)$$
$$= \alpha\sqrt{\gamma}[P^{\ ij}_{(\nu)}K_{ij} - F^{\ j}_{(\nu)}\partial_j \ln\alpha - S^{\ \alpha}_{(\nu)}n_\alpha],$$

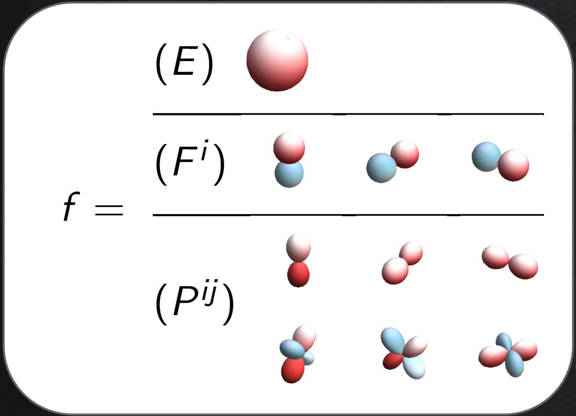
$$\partial_t(\sqrt{\gamma}F_{(\nu)i}) + \partial_j[\sqrt{\gamma}(\alpha P^{\ j}_{(\nu)i} - \beta^j F_{(\nu)i})] - \frac{\partial}{\partial\nu}\Big(\nu\alpha\sqrt{\gamma}\gamma_{i\alpha}M^{\ \alpha\beta\gamma}_{(\nu)}\nabla_\gamma u_\beta\Big)$$
$$= \sqrt{\gamma}\Big[-E_{(\nu)}\partial_i\alpha + F_{(\nu)k}\partial_i\beta^k + \frac{\alpha}{2}P^{\ jk}_{(\nu)}\partial_i\gamma_{jk} + \alpha S^{\ \alpha}_{(\nu)}\gamma_{i\alpha}\Big];$$

Shibata+ (2011)

If the closure is perfect,
The moment equations are exact.

The Moment Closure

$$P^{ij}_{(\nu)} = \frac{3\chi - 1}{2}(P^{ij}_{(\nu)})_{\rm thin} + \frac{3(1-\chi)}{2}(P^{ij}_{(\nu)})_{\rm thick},$$



$$P^{\alpha\beta}_{(\nu)} = E_{(\nu)} \frac{F^{\alpha}_{(\nu)} F^{\beta}_{(\nu)}}{\gamma_{ij} F^{i}_{(\nu)} F^{j}_{(\nu)}}$$

$$L^{\alpha\beta}_{(\nu)} = \frac{1}{3} h^{\alpha\beta} J_{(\nu)}$$

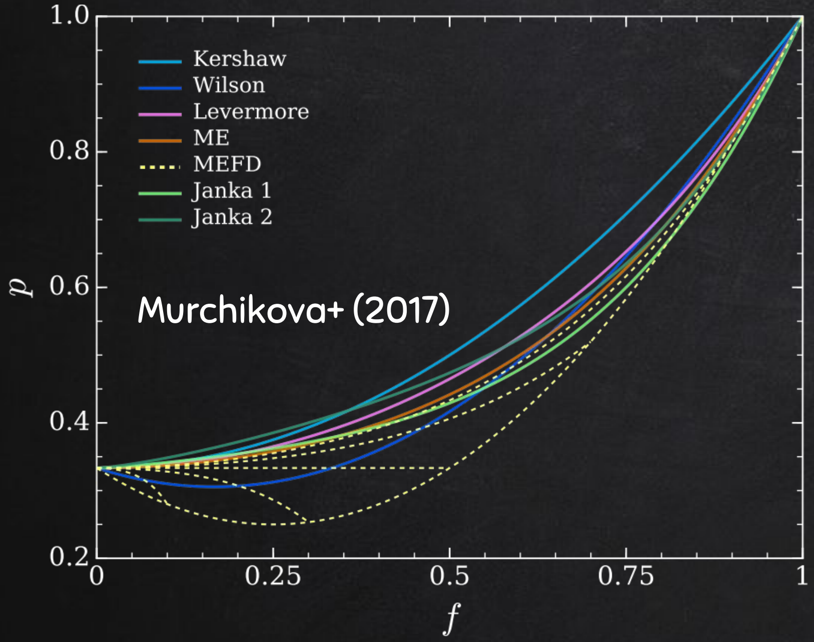
$$\chi = \frac{3 + 4\bar{F}^2}{5 + 2\sqrt{4 - 3\bar{F}^2}}.$$

(Levermore)

$$\bar{F} := \left(\frac{\gamma_{ij} F^{i}_{(\nu)} F^{j}_{(\nu)}}{E^2_{(\nu)}} \right)^{1/2}$$

Guess the pressure tensor using *only* the flux factor.

The Moment Closure



$$P^{ij}_{(\nu)} = \frac{3\chi - 1}{2}(P^{ij}_{(\nu)})_{\text{thin}} + \frac{3(1-\chi)}{2}(P^{ij}_{(\nu)})_{\text{thick}},$$

$$P^{\alpha\beta}_{(\nu)} = E_{(\nu)} \frac{F^{\alpha}_{(\nu)} F^{\beta}_{(\nu)}}{\gamma_{ij} F^{i}_{(\nu)} F^{j}_{(\nu)}}$$

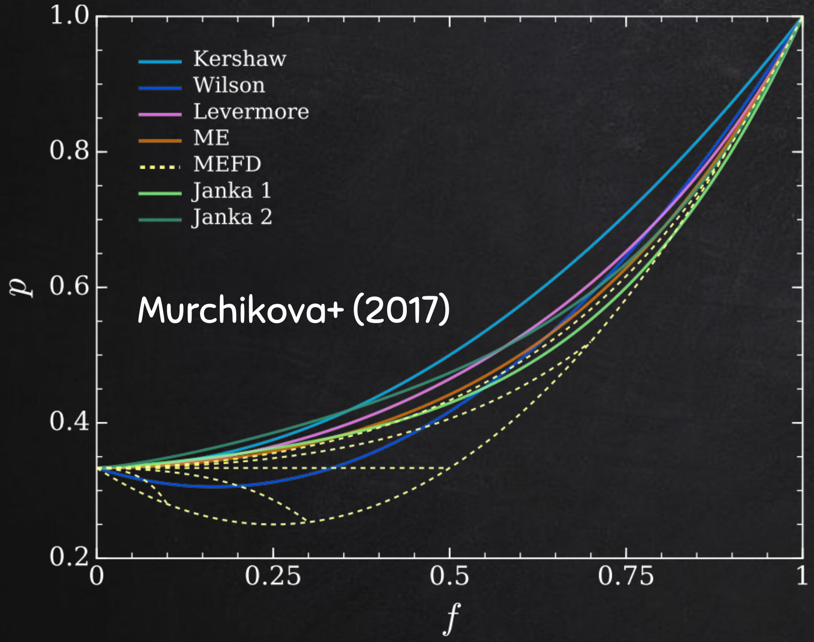
$$L^{\alpha\beta}_{(\nu)} = \frac{1}{3} h^{\alpha\beta} J_{(\nu)}$$

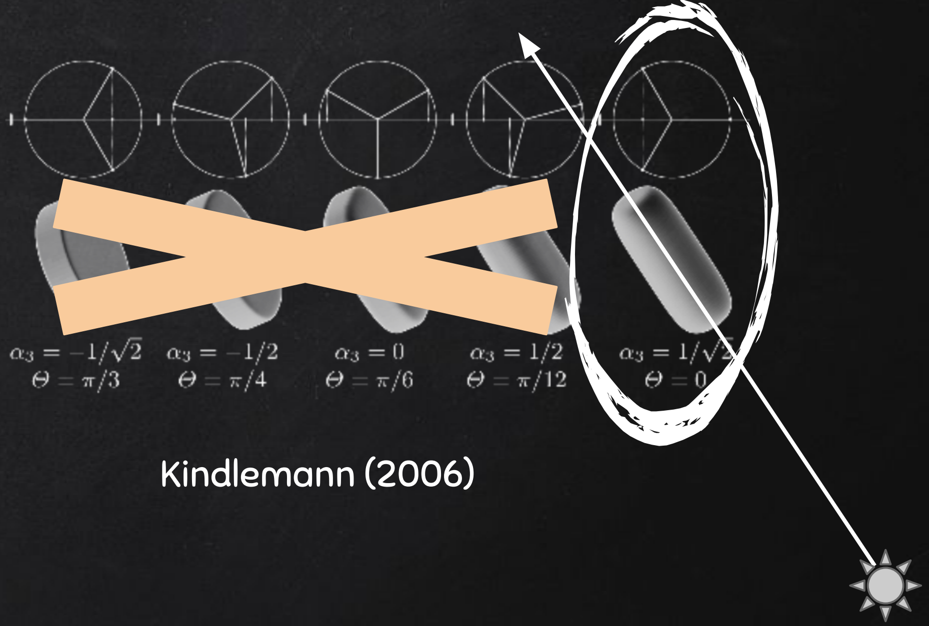
$$\chi = \frac{3 + 4\bar{F}^2}{5 + 2\sqrt{4 - 3\bar{F}^2}}.$$

(Levermore)

$$\bar{F} := \left(\frac{\gamma_{ij} F^{i}_{(\nu)} F^{j}_{(\nu)}}{E^2_{(\nu)}} \right)^{1/2}$$

Pressure Shape an Orientation



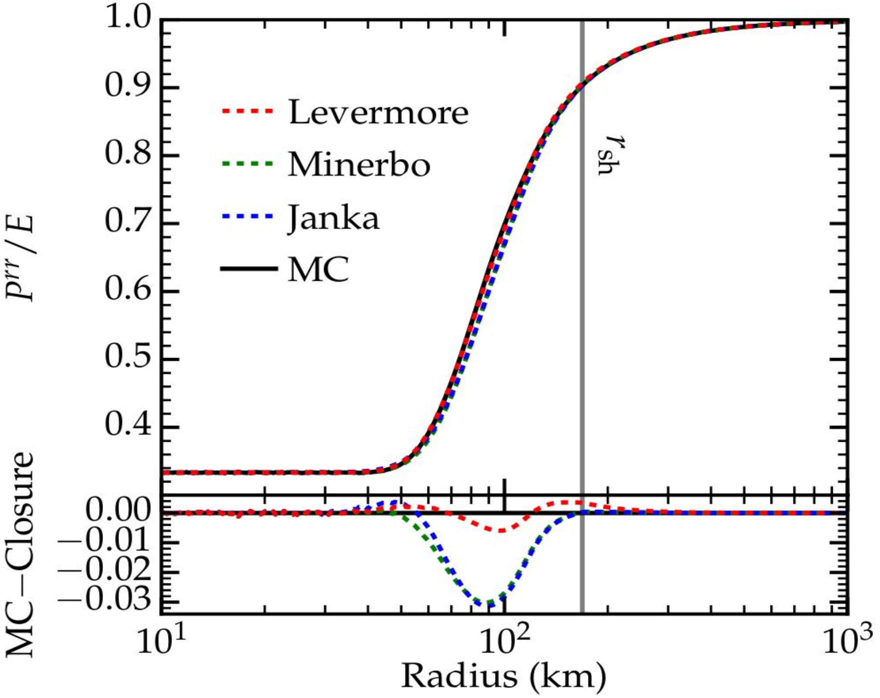


Kindlemann (2006)

<u>Anisotropy:</u> how different is the pressure in different directions?

<u>Oblateness:</u> is the pressure in one direction smaller than the other two directions?

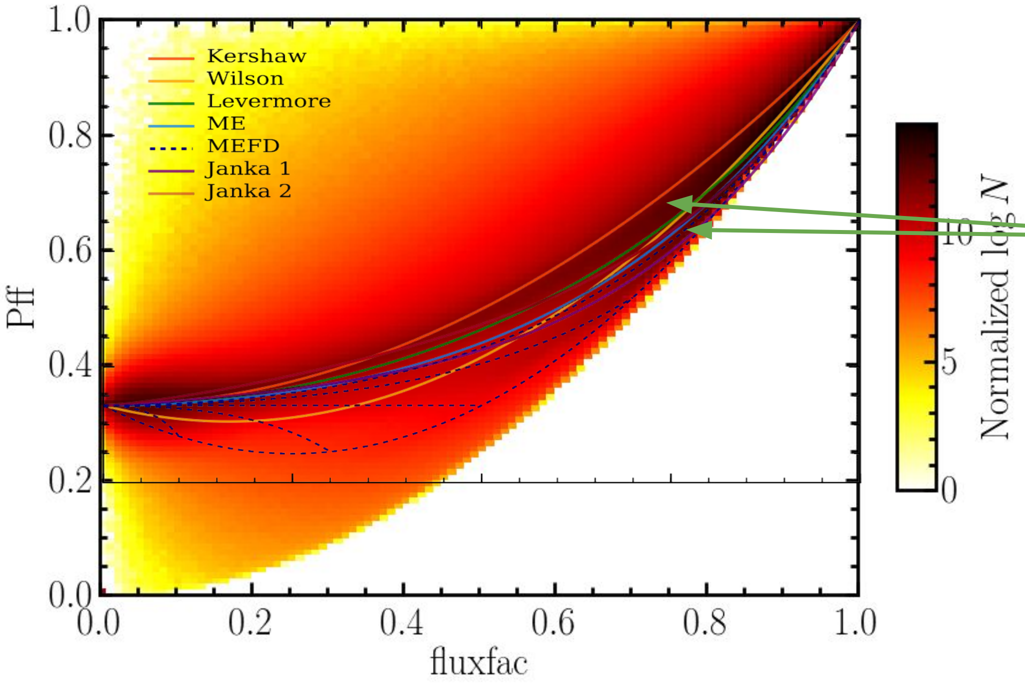
The Moment Closure in 1D



Richers+ (2017)

Assumption 1:

The pressure is a function only of flux

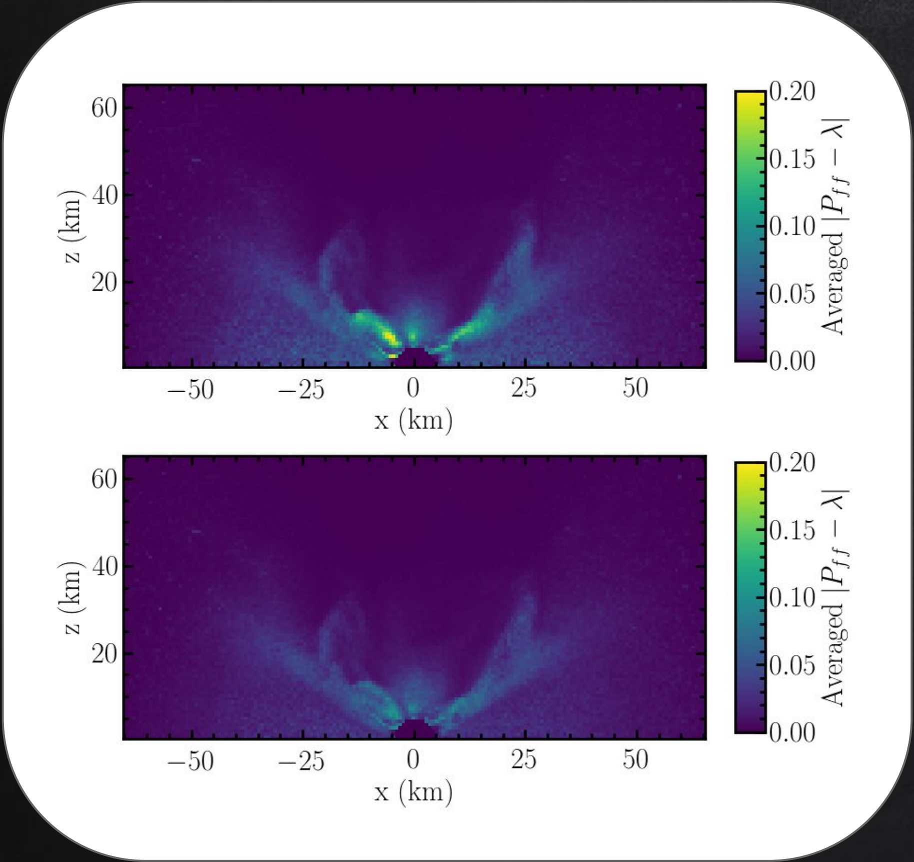


Distinct Branches

No closure fits everything

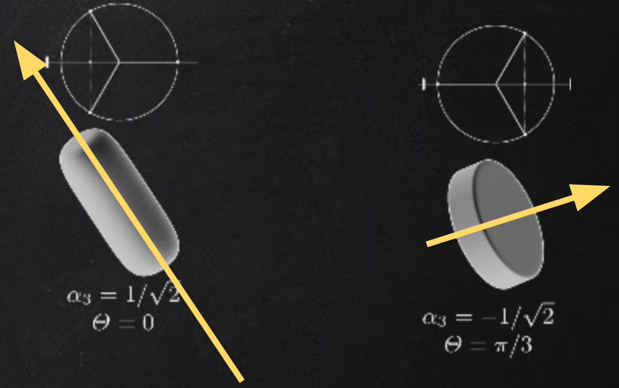
Assumption 2:

Pressure is largest in the direction of flux



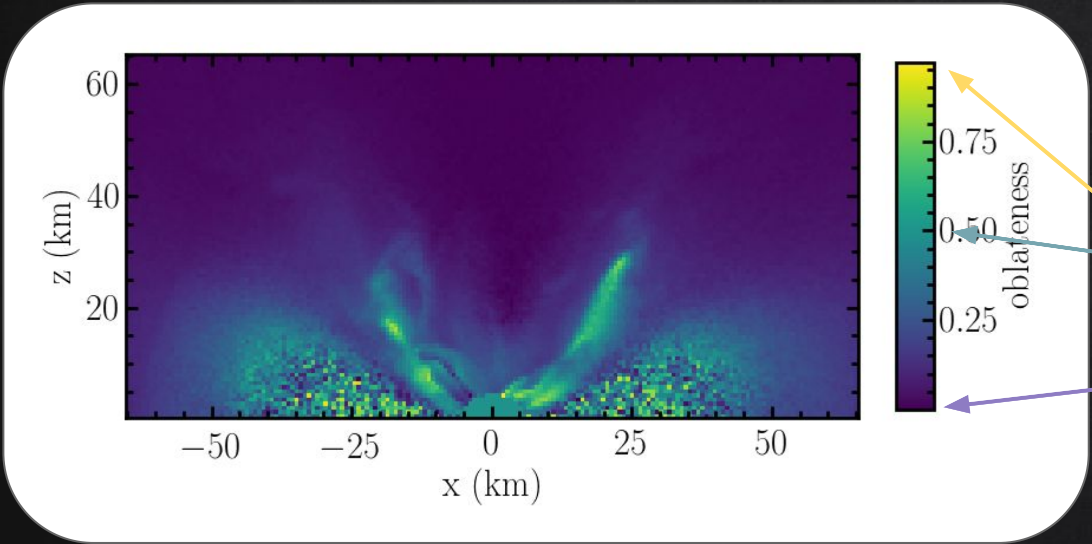
Largest eigenvalue

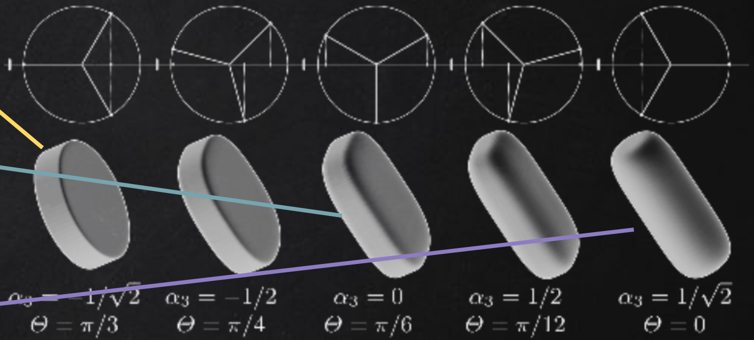
Largest OR smallest eigenvalue



Pressure is largest *or smallest*
in direction of flux
(±10%)

Assumption 3:
The pressure tensor is prolate

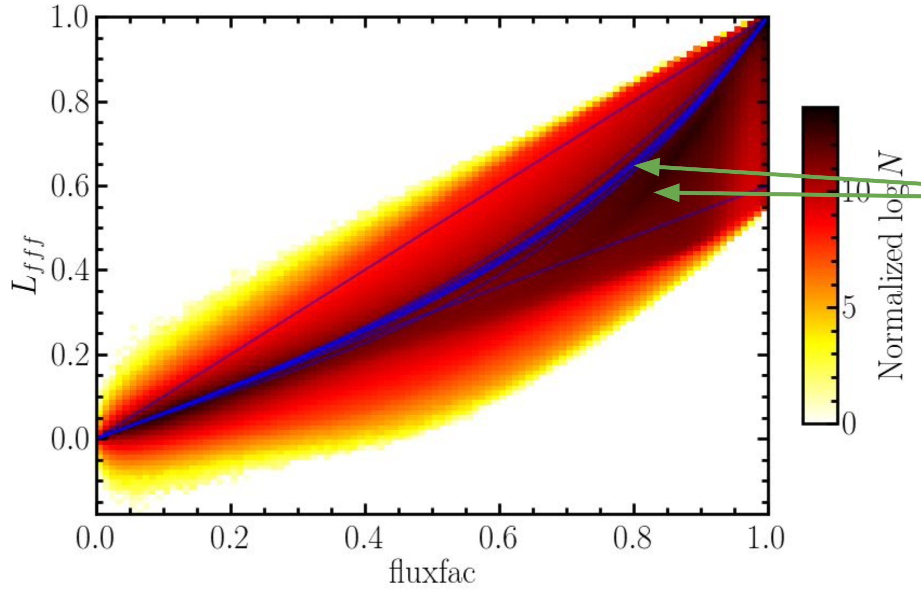




The pressure tensor takes on full range of oblatenesses.

Assumption 4:

The third moment closure is the same



Distinct Branches

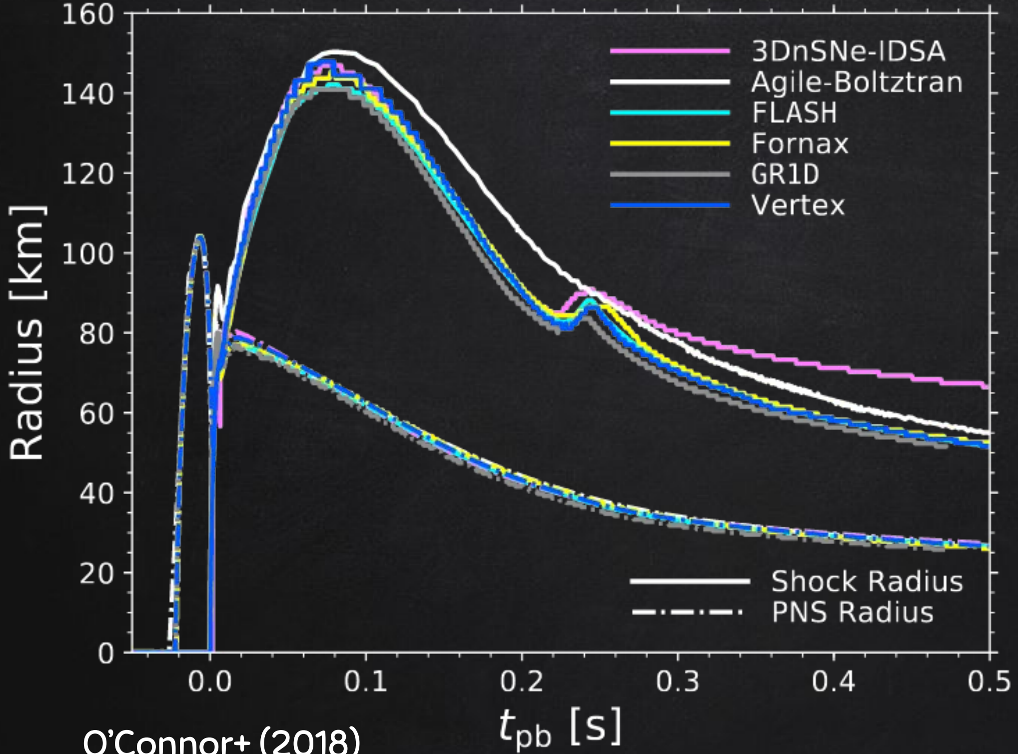
No closure fits everything

The third moment has *the same problems, but not the same closure* as the pressure tensor.

So how can we actually improve closures?

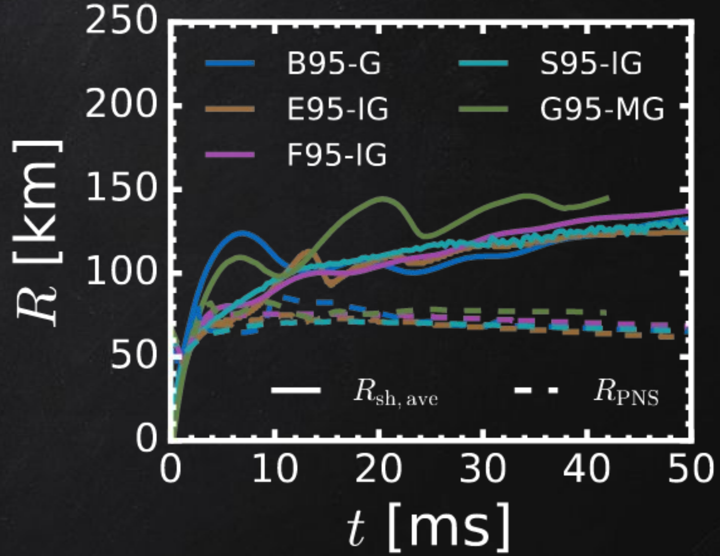
Supernovae will benefit, too!

1D



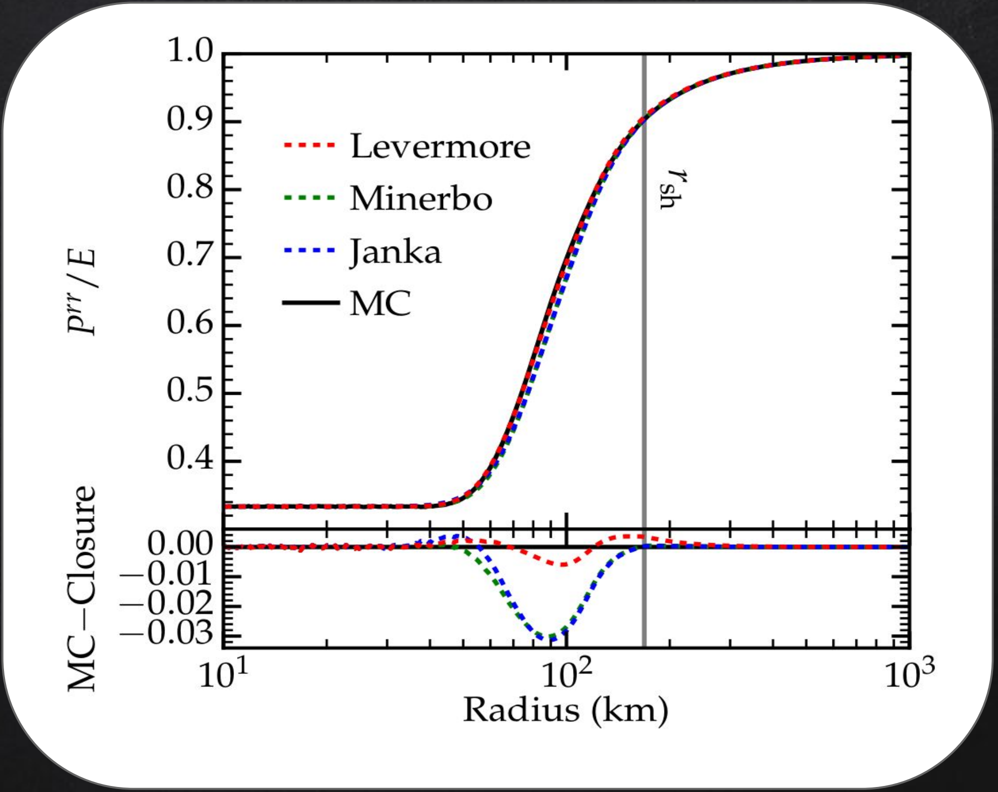
O'Connor+ (2018)

3D



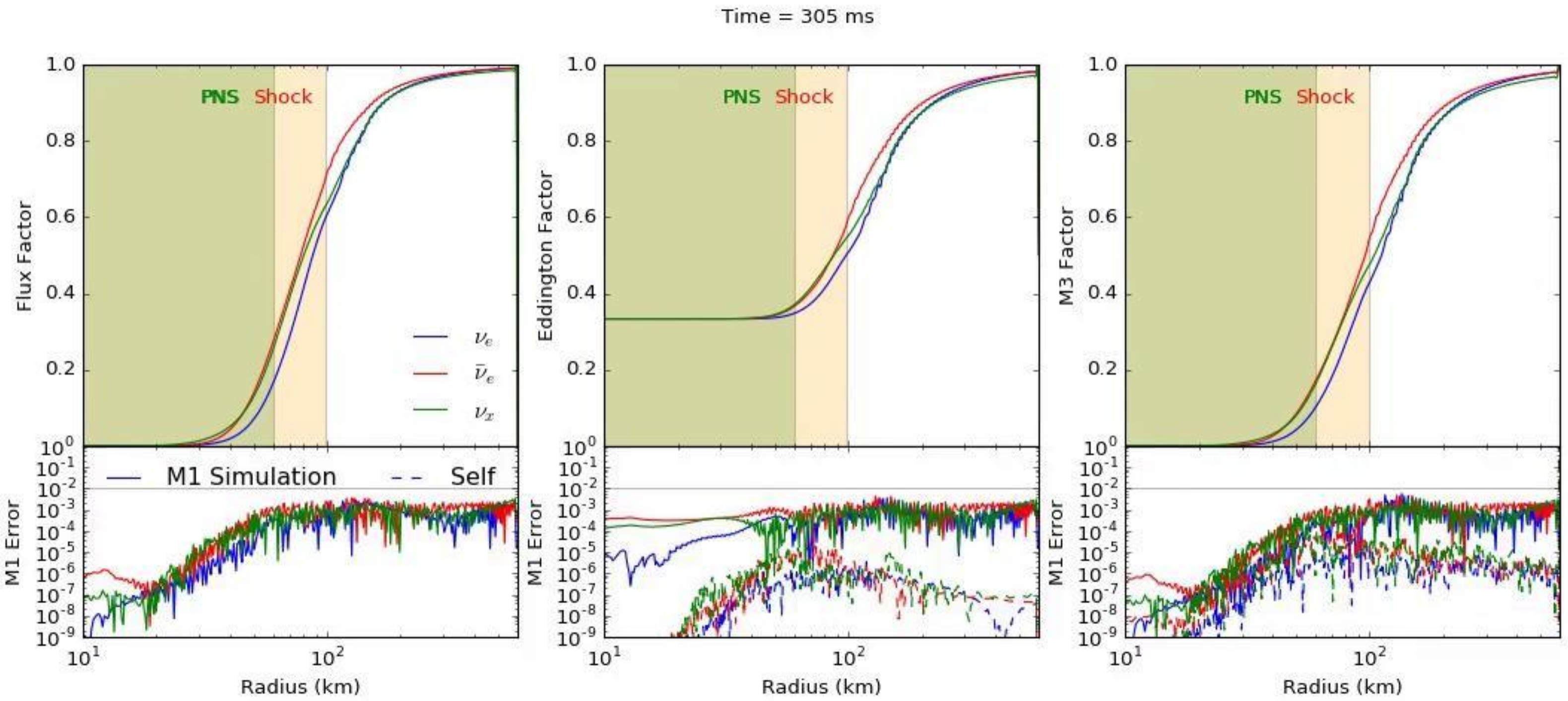
Cabezón+ (2018)

Supernovae will benefit, too!



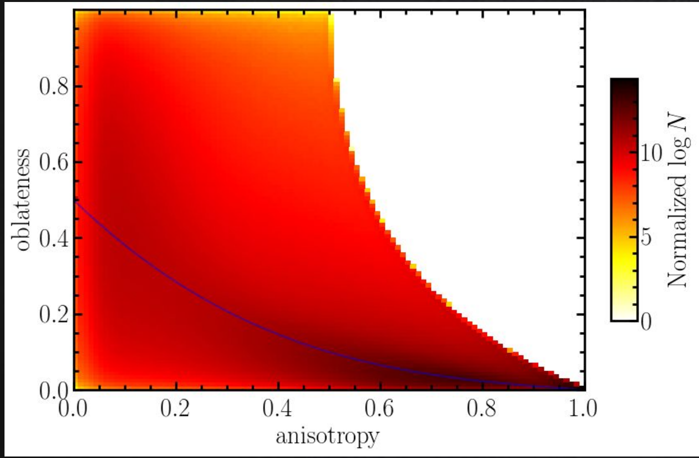
Richers+ (2017)

Supernovae will benefit, too!

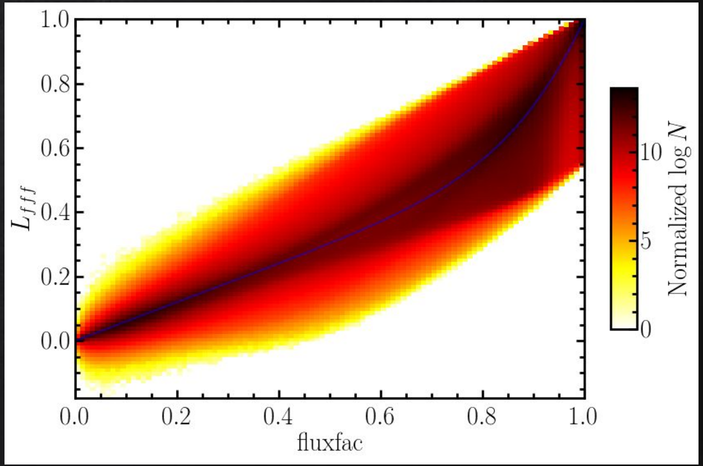


How can we improve closures?

Correlation between anisotropy and oblateness



Separate closure for third moment



How can we improve closures?

Errors before fix:

Closure	P_{ff}	Θ	A	L_{fff}	L_4
thick	0.16	0.35	0.25	0.05	0.031
thin	0.6	0.33	0.85	0.1	0.089
Kershaw	0.051	0.33	0.089	0.038	0.026
Wilson	0.042	0.37	0.078	0.03	0.019
Levermore	0.036	0.33	0.08	0.031	0.018
MEFD	0.033	0.33	0.083	0.029	0.016
MEFD maxpack	0.074	0.39	0.079	0.025	0.015
MEFD classical	0.033	0.33	0.082	0.029	0.016
ME	0.033	0.33	0.082	0.029	0.016
Janka1	0.035	0.33	0.083	0.026	0.014
Janka2	0.038	0.33	0.07	0.03	0.016

Errors after fix:

Closure	P_{ff}	Θ	A
thick	0.16	0.35	0.25
thin	0.41	0.33	0.85
Kershaw	0.047	0.15	0.089
Wilson	0.038	0.15	0.078
Levermore	0.034	0.15	0.08
MEFD	0.033	0.15	0.083
MEFD maxpack	0.059	0.17	0.079
MEFD classical	0.033	0.15	0.082
ME	0.033	0.15	0.082
Janka1	0.036	0.15	0.083
Janka2	0.035	0.15	0.07

L_{fff}	L_4
.022	.014

(some statistical noise still present – doing longer runs)

Conclusions

Moment closures for neutrino transport fail in 3D relativistic environments.

Demonstrated possibility for particle methods to improve the moment closures.

SedonuGR

github.com/srichers/SedonuGR

Visit me on zoom after the talk!